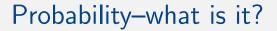
Introduction to Geostatistics

4. Probability I: events, independence, conditional probability

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When there's anything we don't know for sure, we can use probability to describe our knowledge of it. The usual interpretation of probability is that of (limiting) frequency:

- if something happens with probability 0.3, after many repetitions we expect it to happen 30% of the times.
- this implies (assumes) there is some underlying, generating process, which can provide the repetition mechanism.
- sometimes this process is within our control (random sampling), sometimes it is not (and is "nature" said to behave randomly)



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Motivation from philosophy

Ludwig Wittgenstein, Tractatus logico-philosophicus:

- 4.464 A tautology's truth is certain, a proposition's possible, a contradiction's impossible. (Certain, possible, impossible: here we have the first indication of the scale that we need in the theory of probability.)
- 5.153 A proposition is in itself neither probable nor improbable. An event occurs or does not occur, there is no middle course.
- 5.156 It is in this way that probability is a generalization. It involves a general description of a propositional form. We use probability only in default of certainty—if our knowledge of a fact is not indeed complete, but we do know something about its form. ...



Example

- Suppose we have a urn with balls that can be red or blue.
- Person A claims that the urn contains 50% red, and 50% blue balls
- Without looking, we randomly take n = 10 balls out of the urn.
- All of the balls turn out to be red.
- What is the probability that this happens if person A is right?
- Should we believe person A?
- How large should n be (with unicolored or red-only) balls to start disbelieving A?
- Given n, how many balls of the same color do we allow before we reject A's claim?



Looking in detail

- ▶ We have a urn with balls: the population
- we draw one or more balls from it: the sample
- based on the sample, we try to infer properties of the population
- Detail: is every ball taken randomly?
- ► Detail: is every draw independent?
- Independence: drawing with replacement!



Probability as a mathematical construct. Let Pr(E) [S&H: P(E)] be the probability of event E.

We define an event as something that happens, subject to probability. Say, we draw a ball, at random. If A was right, we can say

$$Pr(ball is red) = 0.5$$

- Probability, as a measure, has three properties:
 - 1. The empty (not realizable) event, \emptyset , has probability zero:

$$\Pr(\emptyset) = 0$$

2. Any event has nonnegative probability:

$$\Pr(E) >= 0$$

3. Any possible event has positive probability:

$$\Pr(E) > 0$$



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The R function runif

runif draws one or more random uniform values between 0 and 1:
> runif(1)
[1] 0.844497
> runif(10)
[1] 0.92522851 0.43127488 0.62122534 0.50114290 0.16592987 0.96754708
[7] 0.02003596 0.07540448 0.79720108 0.36114214
> 0.5 < 0.25
[1] FALSE
> runif(5) < 0.25
[1] FALSE FALSE FALSE TRUE FALSE
> runif(5) < 0.25
[1] TRUE FALSE FALSE FALSE FALSE FALSE
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Probability and frequencies

> mean(runif(1) < 0.25)
[1] 0
> mean(runif(10) < 0.25)
[1] 0.2
> mean(runif(100) < 0.25)
[1] 0.29
> mean(runif(1000) < 0.25)
[1] 0.234
> mean(runif(10000) < 0.25)
[1] 0.2469
> mean(runif(1e+05) < 0.25)
[1] 0.25065</pre>



The probability of more than one event

- "AND": $Pr(E_1 \cap E_2)$
- for mutually exclusive events: $Pr(E_1 \cap E_2) = 0$
- "OR": $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) Pr(E_1 \cap E_2)$
- For mutually exclusive, probabilities sum to unity

$$\sum_{i=1}^{n} \Pr(E_i) = 1$$

Summarizing:

$$0 \leq \Pr(E_i) \leq 1$$

 $\Pr(\cup_{all} E_i) = 1$
 $\Pr(\emptyset) = 0$



Conditional probabilities.

The probability of event E_2 , given that E_1 happens, is the so-called conditional probability, and is defined as

$$\Pr(E_2|E_1) = \frac{\Pr(E_2 \cap E_1)}{\Pr(E_1)}$$

we can read it as the probability of E_2 happening, conditional to event E_1 happening.



Conditional probabilities.

> t1 = table(students[c("I.am.2", "Gender")])

> prop.table(t1)

(Gender		
I.am.2	female	male	
small	0.128	0.047	
intermediate	0.163	0.360	
tall	0.093	0.209	
> prop.table(t1, 1)			
(Gender		
I.am.2	female	male	
small	0.73	0.27	
intermediate	0.31	0.69	
tall	0.31	0.69	
> prop.table(t	1, 2)		
Gender			
I.am.2	female	male	

1.am.2	female	male
small	0.333	0.075
intermediate	0.424	0.585
tall	0.242	0.340



Independence

We know that

$$\Pr(E_2|E_1) = \frac{\Pr(E_2 \cap E_1)}{\Pr(E_1)}$$

Two events E_1 and E_2 with $Pr(E_1) > 0$ are said to be *stochastically independent* if

$$\Pr(E_2|E_1) = \Pr(E_2)$$

which implies that

$$\Pr(E_2 \cap E_1) = \Pr(E_1)\Pr(E_2)$$



Independence

```
> ab = rep(c("AA", "BB"), 8)
> cd = rep(c("CC", "DD"), each = 8)
> t1 = table(ab, cd)
> t1
cd
ab CC DD
AA 4 4
BB 4 4
> prop.table(t1, 1)
  cd
ab CC DD
 AA 0.5 0.5
 BB 0.5 0.5
> prop.table(t1, 2)
  cd
ab CC DD
 AA 0.5 0.5
 BB 0.5 0.5
```

